Pricing Single Name Credit Derivatives

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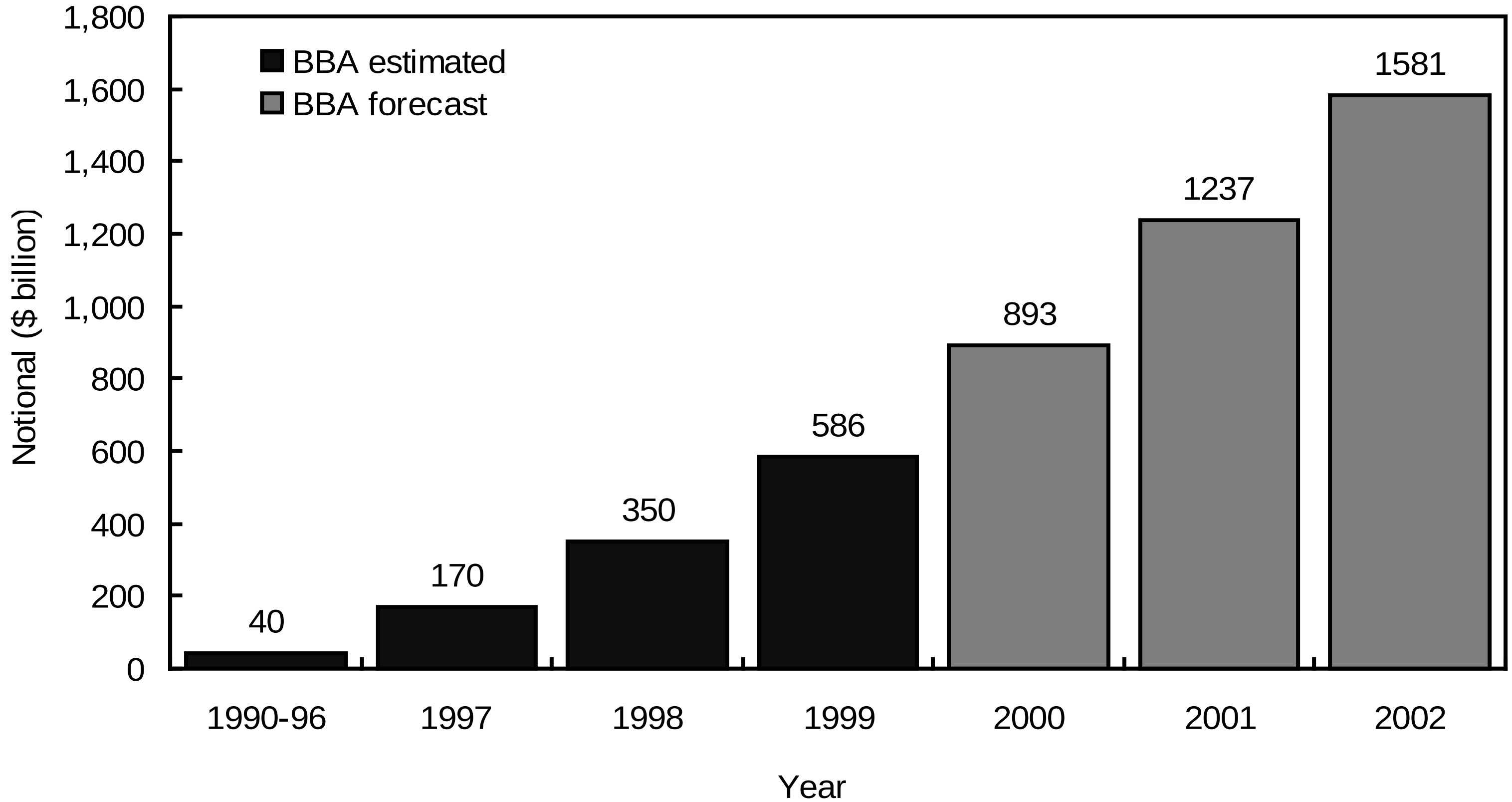
Outline

- Realities of the CDS market
- Pricing Credit Default Swaps
- Generating Clean Risky Discounting Curve
- Effect of Recovery Value
- Hedging CDS
- Pricing Default in Foreign Currency

Credit Derivatives – Size of the Market



- Approximately 40% of the market notional come from Credit Default Swaps

Realities of CDS Market

- Standardized ISDA Credit Derivatives Definitions (1999) provides industry-wide standards and ease of execution
- Two-way Credit Default Swap market in Investment Grade and Emerging Markets, nascent HY CDS market
  - High spread volatility: from 40% up to 300%
  - Risk management with a lack of liquidity:

    Short end of the yield curve Vs. long end

    Gap risk
- Wide range of spreads: from 30 bp to “the sky is the limit”
- Default is not a theoretical possibility but a fact of life (Russia, Ecuador, Laidlaw, etc)
- Reasonably deep cash market with a variety of bonds
- Traded volatility in EM (mostly short maturities)
- Illiquid longer term volatility through options on CDS and Asset Swaps
- Increase in active risk management and more rational credit pricing
- Widespread opportunities to exploit pricing anomalies

Benchmark Curves for a Given Name

- Default-free Discounting Curve (PV of $1 paid with certainty)

$$D(0,t) = E_0\left[\exp\left(-\int_0^t r_\tau d\tau\right)\right] = \exp\left(-\int_0^t \hat{r}_\tau d\tau\right)$$

- Clean Risky Discounting Curve [CRDC] (PV of $1 paid contingent on no default till maturity, otherwise zero)

$$Z\ (0,t) = E_0\left[\exp\left(-\int_0^t (r_\tau + \lambda_\tau) d\tau\right)\right]$$

$\lambda_\tau d\tau$ has a meaning of default probability at time τ over time period $d\tau$

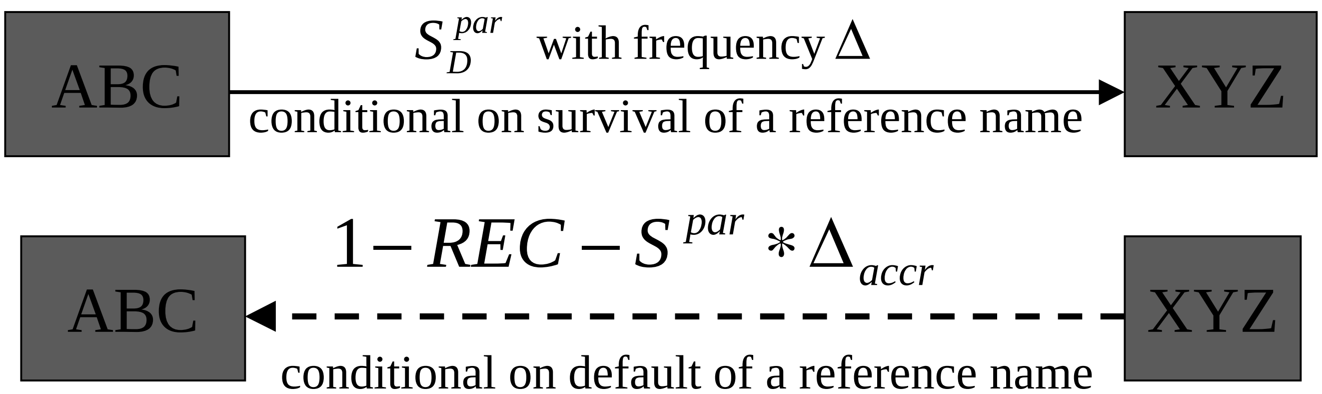
$$Z\ (0,t) = D(0,t) * Q(0,t)$$

$$Q(0,t) = \exp\left(-\int_0^t \hat{\lambda}_\tau d\tau\right)$$

where $Q(0,t)$ is survival probability till time t, and $\hat{\lambda}_\tau$ is usually interpreted as forward (not expected !) probability of default per unit time

Credit Default Swap

- A basic credit derivatives instrument: ABC is long default protection



- REC is a recovery value of a reference bond
- Reference bond: no guarantied cash flows
  cheapest-to-deliver
  cross-default (cross-acceleration)
- Assume same recovery value REC for all CDS of the same seniority on a given name

Pricing CDS

- For corporate and EM coupon bonds a default claim is (Principal + Accrued Interest)
- Recovery value has very little sensitivity to a structure of bond cash flows
- For this Face Value Claim, REC = R , and PV of CDS is given by

$$PV_{CDS} = -S_T E_0\left[\int_0^T e^{-\int_0^t (r_\tau+\lambda_\tau)d\tau} dt\right] + E_0\left[\int_0^T (1-R_t)\lambda_t e^{-\int_0^t (r_\tau+\lambda_\tau)d\tau} dt\right] \quad (1)$$

- Assume $R_t = R, \quad \overline{R} = E(R)$, and no correlation of R with spreads and interest rates
- As Eq (1) is linear in R, CRDC just depends on expected value $\overline{R}$, not on distribution of R
- Put PV of CDS = 0, and bootstrapping allows us to generate a clean risky discounting curve

Generating CRDC

- A term structure of par credit spreads $S_{T,par}$ is given by the market
- To generate CRDC we need to price both legs of a swap
- No Default (fee) leg

$$S_{T,par} E_0 \left[\int_0^T e^{-\int_0^t (r_\tau + \lambda_\tau) d\tau} dt \right] = S_{T,par} \int_0^T Z\ \ (0,t) dt$$

- Default leg

$$E_0 \left[\int_0^T (1 - R_t) \lambda_t e^{-\int_0^t (r_\tau + \lambda_\tau) d\tau} dt \right] = (1 - \overline{R})\, E_0 \left[\int_0^T \lambda_t e^{-\int_0^t (r_\tau + \lambda_\tau) d\tau} dt \right] = (1 - \overline{R}) \int_0^T \tilde{\lambda}_t Z\ \ (0,t) dt$$

- If correlation between credit spreads and interest rates is not zero,

$$\tilde{\lambda}_\tau \neq \hat{\lambda}_\tau$$

Correlation Adjustment

- Need to take into account correlation between spreads and interest rates to calculate adjusted forward default probability $\tilde{\lambda} = \hat{\lambda} + a_\lambda$

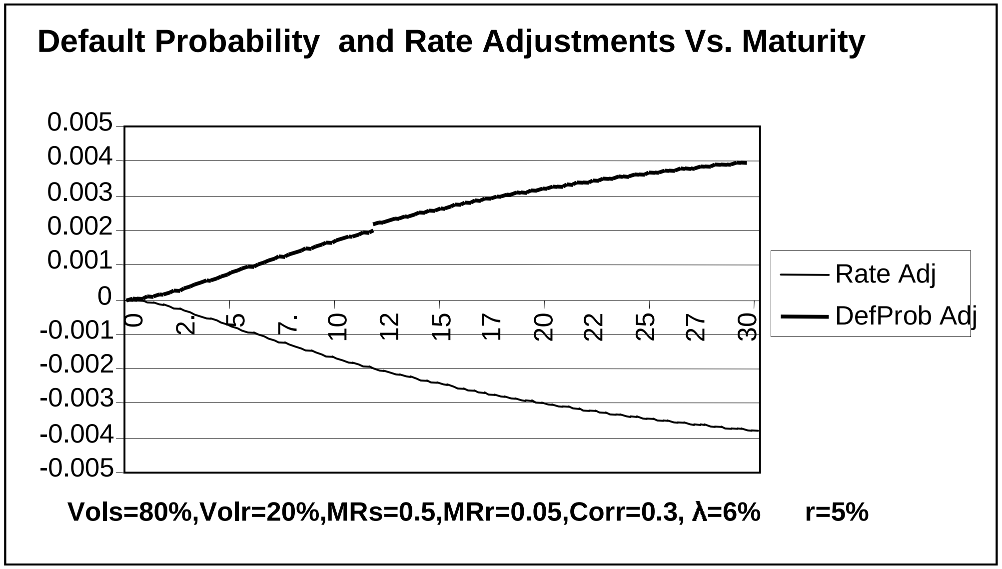
Default-free rate $\tilde{r}$ conditional on no default also needs to be adjusted as $\tilde{r} = \hat{r} - a_\lambda$

$$-\frac{dZ(0,t)}{dt}$$

$$= E_0\left[(r_t + \lambda_t)\exp\left(-\int_0^t (r_\tau + \lambda_\tau) d\tau \right) \right]$$

and

$$\tilde{s} + \tilde{r} = \hat{s} + \hat{r}$$



- For high spreads and high volatilities an adjustment is not negligible
- For given par spreads forward default probability decreases with increasing volatility, correlation and level of interest rates and par spreads

Recovery Value

- For the Face Value Claim the price of a generic coupon bond can be approximated pretty accurately as

$$B(t,t_N) = \sum_n C_n Z \quad (t,t_n) + Z \ (t,t_N)$$
$$+\overline{R}\left[1-\sum_n^N \Delta_n \tilde{L}_{n-1} Z \ (t,t_n) - Z \ (t,t_N)\right]$$

where $\tilde{L}_{n-1}$ is forward default-free floating rate for period n

- Bond price goes to recovery value in default
- For the same default risk and recovery value, high coupon bond should trade at higher credit spread than a low coupon bond
- There are no generic risky zero coupon bonds with non zero recovery
- Using CRDC and given recovery value structure one can create any synthetic instrument

More on Recovery Value

- Other ways to model recovery value:
  - Recovery of the Risky Price (Duffie-Singelton) :

    Default claim is a traded price just before the default event
  - Recovery of the Riskless Price:

    Default claim is given by default-free PV of the bond cash flows at the moment of default

    For a zero coupon bond this default claim corresponds to the claim on a face value at maturity
  - Both methods operate with risky zero coupon bonds with embedded recovery values. One can use conventional bond math for risky bonds

- Both methods are not applicable in the real markets

Effect of Recovery Value Assumptions on Relative Value

- Implications for pricing off-market deals, synthetic instruments, risk management
- Example: Relative bond value

  Same name, seniority and maturity, different coupons
- For Recovery of Face Value

$$B_C - B_{C'} = \sum_{n=1}^{N} (C_n - C'_n) D(0,t_n) Q(0,t_n)$$

- For Recovery of Risky/Riskless Price

$$B_C - B_{C'} = \sum_{n=1}^{N} (C_n - C'_n) Z_R(t,t_n)$$

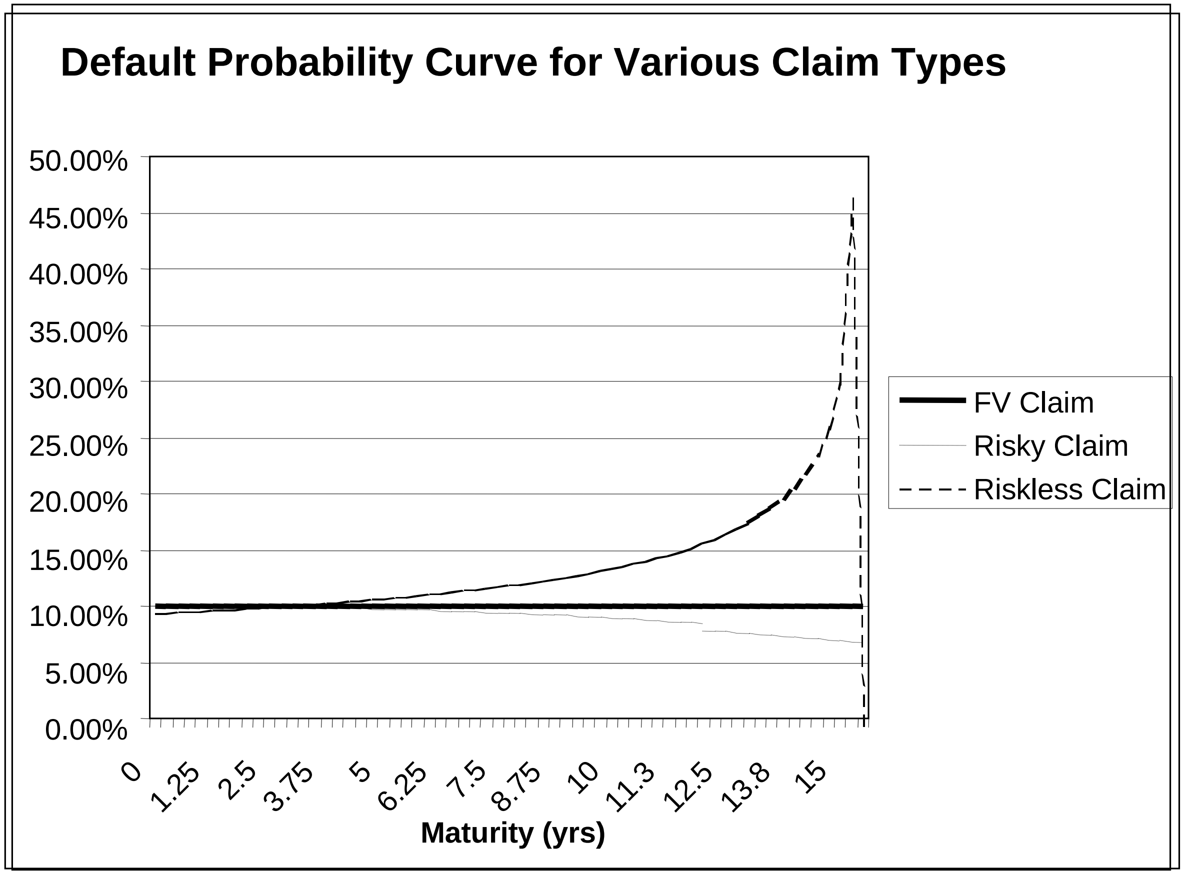
Risky zero coupon bonds with embedded recovery value are given by

$$Z_R(0,t) = E\left\{ \exp\left[-\int_0^t \left(r_\tau + (1-R)\lambda_\tau \right) d\tau \right] \right\} \approx D(0,t) Q(0,t)^{(1-\overline{R})}$$ for Risky Claim

$$Z_R(0,t) = D(0,t)\left[(1-\overline{R}) Q(0,t) + \overline{R} \right]$$ for Riskless Claim

Default Probability and Recovery

- For a given par credit spread curve default probabilities depend on recovery value definition



- Par Spreads = 6%
- Volatility = 40%
- R = 0.4

Hedging CDS books

- Two types of exposures: credit spread risk, default risk
- Using N hedging instruments (bonds or CDS) on can hedge a CDS portfolio against (N-1) predetermined factors for spread moves + default
- Different Recovery Value definitions result in different hedging positions
- Robustness of hedging depends on spread curve interpolation method
- Transaction cost may be significant: need to optimize hedging strategy
- When hedging with bonds, bond/CDS basis risk can be an issue
- In EM cheapest-to-deliver option is equivalent to first-to-default feature
- For HY CDS equity options/shares should be considered as possible hedging instrument

Pricing Default in Foreign Currency

- Assume that one needs to sell default protection in foreign currency and hedge it by buying protection in $.

  Q: At what level to sell?

- If there is no interdependence between credit spreads and forward FX, implied default probabilities should stay the same in foreign currency

- Due to the correlation between default spread and each of FX, dollar interest rates, and foreign interest rates, the default probability in a foreign currency will differ from that in dollars

- Two sources for the adjustment:
  - Devaluation conditional on default
  - Day-to-day spread/FX/IR correlation

Adjustment for FX jump conditional on Default

- FX rate jumps by -α % when default occurs (e.g. devaluation)
- As probability of default (and FX jump) is given by λ_t , under "no default" conditions the foreign currency (FC) should have an excessive return in terms of USD (DC) given by $\lambda_t * \alpha$ to compensate for a possible loss of value
- Consider a FC clean risky zero coupon bond (R=0) with an excessive "no default" return λ_t^F that compensates for a possible default
- The position value in DC = (Bond Price in FC) * (Price of FC in DC)
- An excessive return of the position in DC is $\lambda_t * \alpha + \lambda_t^F$
- The position should have the same excessive return as any other risky bond in DC which is given by λ_t
- To avoid arbitrage the FC credit spread should be

$$\lambda_t^F = \lambda_t * (1 - \alpha)$$

- An adjustment can be substantial

Quanto Spread Adjustment

- In the no default state, correlation between FX rate and interest rates on one side and the credit spread on another results in a quanto adjustment to the credit spread curve used to price a synthetic note in FC

- Consider hedges for a short in synthetic risky note in FC
  - sell default protection in DC
  - long FC, short DC

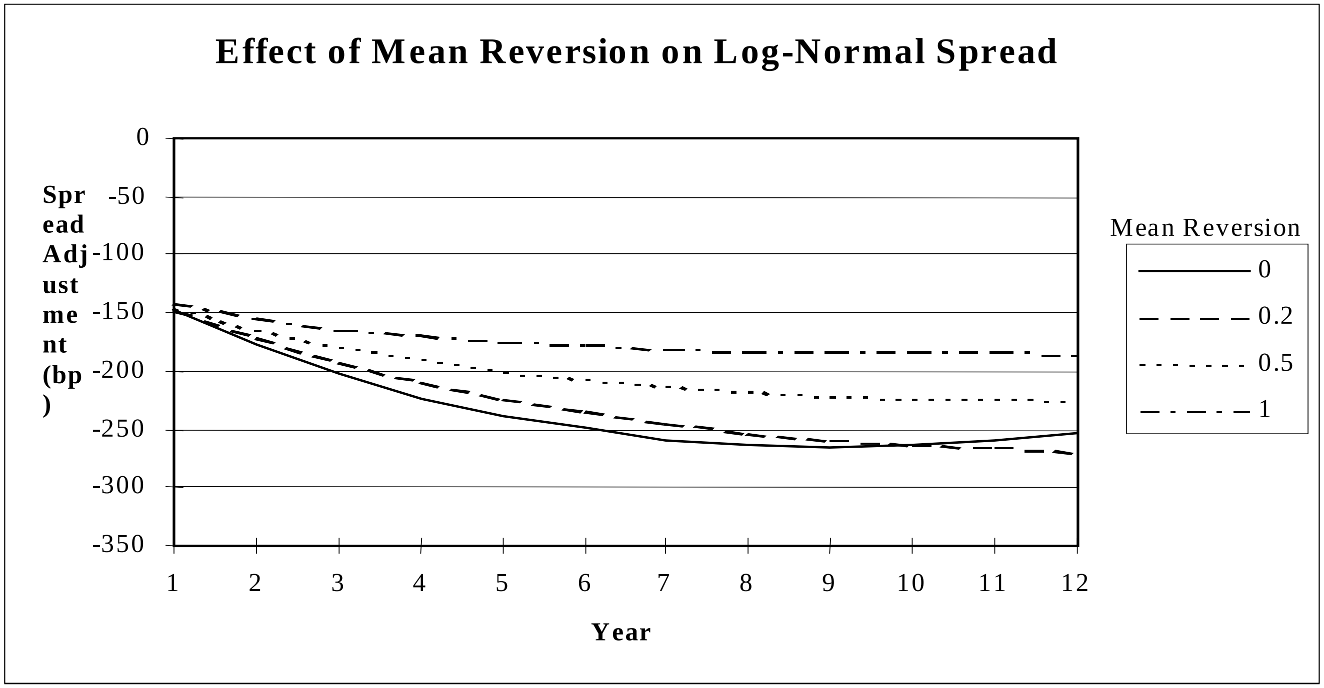
- If DC strengthens as spreads widen, in order to hedge the note we would need to buy back some default protection and sell the foreign currency that depreciated.

- Our P&L would suffer and we would need to pass this additional expense to a counter party in a form of a negative credit spread adjustment

- For high correlation the adjustment can be significant

Quanto Adjustment (cont'd)

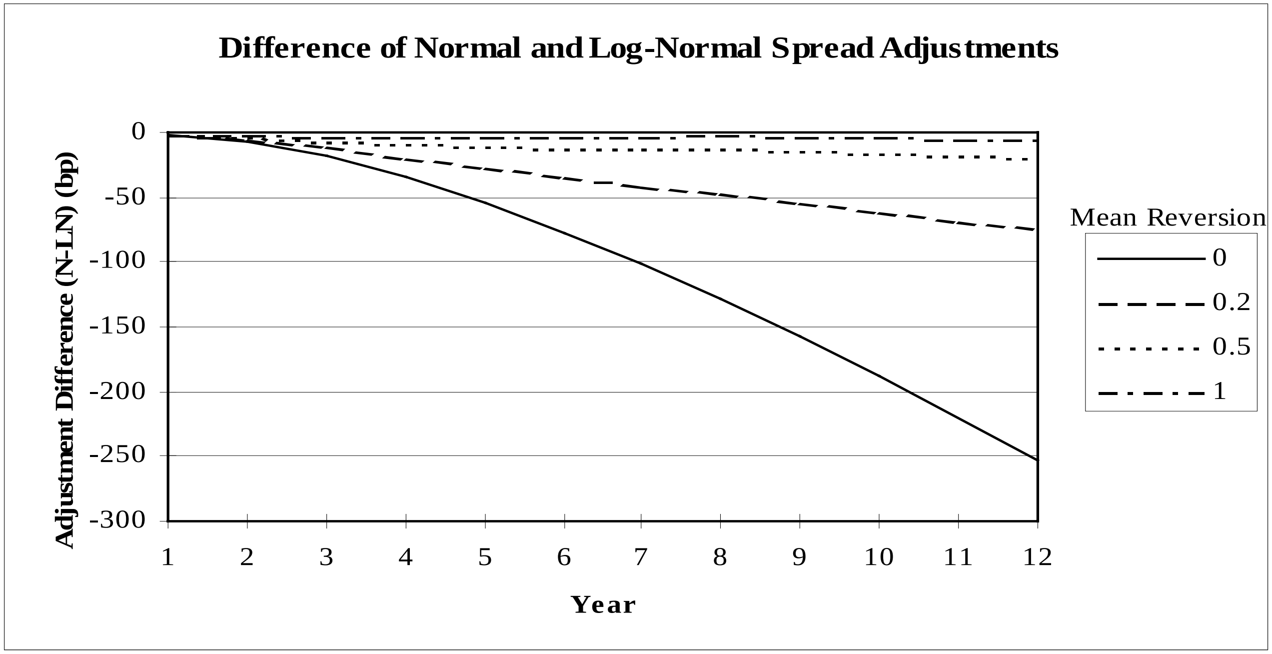
- Adjustment for a DC flat spread curve of 600 bp. Spread MR is important



- Spread adjustment decreases with increasing mean reversion and constant spot volatility. α =0.2, S=6%, r\$=5%, rf=20%, σs=80%, σ\$=12.5%, β\$= 0, σf=40%, βs=0.5, σx=20%, ρ\$s=0, ρfs=0.5, ρxs=0.7. All curves are flat.

Quanto Adjustment (cont'd)

- Assumptions on spread distribution are important



- Difference between normal and log-normal adjustment decreases as mean reversion is increased for constant spot volatility

Conclusions

- For single-name instruments pricing is well understood
- Recovery value definition can have an significant effect on pricing and hedging
- Hedging CDS with bonds: basis risk cannot be ignored
- The distinction between EM and FI credit derivatives gradually disappears
- Consistency of pricing and hedging methods becomes more and more important