Pricing Single Name Credit Derivatives

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Outline

- Realities of the CDS market
- Pricing Credit Default Swaps
- Generating Clean Risky Discounting Curve
- Effect of Recovery Value
- Hedging CDS
- Pricing Default in Foreign Currency
Credit Derivatives – Size of the Market

- Approximately 40% of the market notional come from Credit Default Swaps
Realities of CDS Market

- Standardized ISDA Credit Derivatives Definitions (1999) provides industry-wide standards and ease of execution
- Two-way Credit Default Swap market in Investment Grade and Emerging Markets, nascent HY CDS market
  - High spread volatility: from 40% up to 300%
  - Risk management with a lack of liquidity:
    Short end of the yield curve Vs. long end
    Gap risk
- Wide range of spreads: from 30 bp to “the sky is the limit”
- Default is not a theoretical possibility but a fact of life (Russia, Ecuador, Laidlaw, etc)
- Reasonably deep cash market with a variety of bonds
- Traded volatility in EM (mostly short maturities)
- Illiquid longer term volatility through options on CDS and Asset Swaps
- Increase in active risk management and more rational credit pricing
- Widespread opportunities to exploit pricing anomalies
Benchmark Curves for a Given Name

- Default-free Discounting Curve (PV of $1 paid with certainty)

\[ D(0,t) = E_0 \left[ \exp \left( - \int_0^t r_\tau d\tau \right) \right] = \exp \left( - \int_0^t \hat{r}_\tau d\tau \right) \]

- Clean Risky Discounting Curve [CRDC] (PV of $1 paid contingent on no default till maturity, otherwise zero)

\[ Z(0,t) = E_0 \left[ \exp \left( - \int_0^t (r_\tau + \lambda_\tau) d\tau \right) \right] \]

\( \lambda_\tau d\tau \) has a meaning of default probability at time \( \tau \) over time period \( d\tau \)

\[ Z(0,t) = D(0,t) \ast Q(0,t) \]

\[ Q(0,t) = \exp \left( - \int_0^t \hat{\lambda}_\tau d\tau \right) \]

where \( Q(0,t) \) is survival probability till time t, and \( \hat{\lambda}_\tau \) is usually interpreted as forward (not expected!) probability of default per unit time
Credit Default Swap

- A basic credit derivatives instrument: ABC is long default protection

\[ S^\text{par}_D \text{ with frequency } \Delta \]
conditional on survival of a reference name

\[ 1 - \text{REC} - S^\text{par}_D \ast \Delta_{\text{accr}} \]
conditional on default of a reference name

- REC is a recovery value of a reference bond
- Reference bond: no guarantied cash flows
cheapest-to-deliver
cross-default (cross-acceleration)
- Assume same recovery value REC for all CDS of the same seniority
on a given name
Pricing CDS

- For corporate and EM coupon bonds a default claim is (Principal + Accrued Interest)
- Recovery value has very little sensitivity to a structure of bond cash flows
- For this Face Value Claim, \( \text{REC} = R \), and PV of CDS is given by

\[
P_{\text{CDS}} = -S_T E_0 \left[ \int_0^T e^{-\int_0^t (r_\tau + \lambda_\tau) d\tau} \, dt \right] + E_0 \left[ \int_0^T (1 - R_t) \lambda_t e^{-\int_0^t (r_\tau + \lambda_\tau) d\tau} \, dt \right]
\]

(1)

- Assume \( R_t = R \), \( \bar{R} = E(R) \), and no correlation of R with spreads and interest rates
- As Eq (1) is linear in R, CRDC just depends on expected value \( \bar{R} \), not on distribution of R
- Put PV of CDS = 0, and bootstrapping allows us to generate a clean risky discounting curve
Generating CRDC

- A term structure of par credit spreads $S_{T, par}$ is given by the market.
- To generate CRDC we need to price both legs of a swap.
- No Default (fee) leg

$$S_{T, par} E_0 \left[ \int_0^T e^{-\int_0^t (r_{\tau} + \lambda_{\tau}) d\tau} dt \right] = S_{T, par} \int_0^T Z(0,t) dt$$

- Default leg

$$E_0 \left[ \int_0^T (1 - R_t) \lambda_t e^{-\int_0^t (r_{\tau} + \lambda_{\tau}) d\tau} dt \right] = (1 - \bar{R}) E_0 \left[ \int_0^T \lambda_t e^{-\int_0^t (r_{\tau} + \lambda_{\tau}) d\tau} dt \right] = (1 - \bar{R}) \int \tilde{\lambda}_t Z(0,t) dt$$

- If correlation between credit spreads and interest rates is not zero,

$$\tilde{\lambda}_{\tau} \neq \hat{\lambda}_{\tau}$$
Correlation Adjustment

- Need to take into account correlation between spreads and interest rates to calculate adjusted forward default probability

\[ \lambda = \lambda + a_\lambda \]

Default-free rate \( \tilde{r} \) conditional on no default also needs to be adjusted as

\[ \tilde{r} = \hat{r} - a_\lambda \]

\[
- \frac{dZ(0,t)}{dt} \]

\[ = E_0 \left[ (r_t + \lambda_t) \exp \left( - \int_0^t (r_\tau + \lambda_\tau) d\tau \right) \right] \]

and

\[ \tilde{s} + \tilde{r} = \hat{s} + \hat{r} \]

- For high spreads and high volatilities an adjustment is not negligible
- For given par spreads forward default probability decreases with increasing volatility, correlation and level of interest rates and par spreads
Recovery Value

- For the Face Value Claim the price of a generic coupon bond can be approximated pretty accurately as

\[ B(t, t_N) = \sum_n C_n Z(t, t_n) + Z(t, t_N) + \bar{R} \left[ 1 - \sum_n \Delta_n \tilde{L}_{n-1} Z(t, t_n) - Z(t, t_N) \right] \]

where \( \tilde{L}_{n-1} \) is forward default-free floating rate for period n

- Bond price goes to recovery value in default

- For the same default risk and recovery value, high coupon bond should trade at higher credit spread than a low coupon bond

- There are no generic risky zero coupon bonds with non zero recovery

- Using CRDC and given recovery value structure one can create any synthetic instrument
More on Recovery Value

• Other ways to model recovery value:
  – Recovery of the Risky Price (Duffie-Singelton):
    Default claim is a traded price just before the default event
  – Recovery of the Riskless Price:
    Default claim is given by default-free PV of the bond cash flows at the moment of default
    For a zero coupon bond this default claim corresponds to the claim on a face value at maturity
  – Both methods operate with risky zero coupon bonds with embedded recovery values. One can use conventional bond math for risky bonds

• Both methods are not applicable in the real markets
Effect of Recovery Value Assumptions on Relative Value

• Implications for pricing off-market deals, synthetic instruments, risk management
• Example: Relative bond value
  
  Same name, seniority and maturity, different coupons

• For Recovery of Face Value
  \[
  B_C - B_{C'} = \sum_{n=1}^{N} (C_n - C'_n) D(0, t_n) Q(0, t_n)
  \]

• For Recovery of Risky/Riskless Price
  \[
  B_C - B_{C'} = \sum_{n=1}^{N} (C_n - C'_n) Z_R(t, t_n)
  \]

Risky zero coupon bonds with embedded recovery value are given by

\[
Z_R(0, t) = E \left\{ \exp \left[ - \int_0^t \left( r_\tau + (1 - R) \lambda_\tau \right) d \tau \right] \right\} \approx D(0, t) Q(0, t)^{(1 - \overline{R})}
\]

for Risky Claim

\[
Z_R(0, t) = D(0, t) \left[ (1 - \overline{R}) Q(0, t) + \overline{R} \right]
\]

for Riskless Claim
Default Probability and Recovery

- For a given par credit spread curve default probabilities depend on recovery value definition
- Par Spreads = 6%
- Volatility = 40%
- $R = 0.4$

![Default Probability Curve for Various Claim Types](image)
Hedging CDS books

- Two types of exposures: credit spread risk, default risk

- Using N hedging instruments (bonds or CDS) can hedge a CDS portfolio against (N-1) predetermined factors for spread moves + default

- Different Recovery Value definitions result in different hedging positions

- Robustness of hedging depends on spread curve interpolation method

- Transaction cost may be significant: need to optimize hedging strategy

- When hedging with bonds, bond/CDS basis risk can be an issue

- In EM cheapest-to-deliver option is equivalent to first-to-default feature

- For HY CDS equity options/shares should be considered as possible hedging instrument
Pricing Default in Foreign Currency

• Assume that one needs to sell default protection in foreign currency and hedge it by buying protection in $.
Q: At what level to sell?

• If there is no interdependence between credit spreads and forward FX, implied default probabilities should stay the same in foreign currency.

• Due to the correlation between default spread and each of FX, dollar interest rates, and foreign interest rates, the default probability in a foreign currency will differ from that in dollars.

• Two sources for the adjustment:
  - Devaluation conditional on default
  - Day-to-day spread/FX/IR correlation
Adjustment for FX jump conditional on Default

- FX rate jumps by $-\alpha \%$ when default occurs (e.g. devaluation)
- As probability of default (and FX jump) is given by $\lambda_t$, under “no default” conditions the foreign currency (FC) should have an excessive return in terms of USD (DC) given by $\lambda_t \times \alpha$ to compensate for a possible loss of value.
- Consider a FC clean risky zero coupon bond ($R=0$) with an excessive “no default” return $\lambda_t^F$ that compensates for a possible default.
- The position value in DC = (Bond Price in FC) * (Price of FC in DC).
- An excessive return of the position in DC is $\lambda_t \times \alpha + \lambda_t^F$.
- The position should have the same excessive return as any other risky bond in DC which is given by $\lambda_t$.
- To avoid arbitrage the FC credit spread should be

\[ \lambda_t^F = \lambda_t \times (1 - \alpha) \]

- An adjustment can be substantial.
Quanto Spread Adjustment

• In the no default state, correlation between FX rate and interest rates on one side and the credit spread on another results in a quanto adjustment to the credit spread curve used to price a synthetic note in FC

• Consider hedges for a short in synthetic risky note in FC
  – sell default protection in DC
  – long FC, short DC

• If DC strengthens as spreads widen, in order to hedge the note we would need to buy back some default protection and sell the foreign currency that depreciated.

• Our P&L would suffer and we would need to pass this additional expense to a counter party in a form of a negative credit spread adjustment

• For high correlation the adjustment can be significant
Quanto Adjustment (cont’d)

- Adjustment for a DC flat spread curve of 600 bp. Spread MR is important

![Effect of Mean Reversion on Log-Normal Spread](image)

- Spread adjustment decreases with increasing mean reversion and constant spot volatility. \( \alpha = 0.2, S=6\%, r=20\%, s=80\%, \sigma s=12.5\%, \beta s=0, \sigma f=40\%, \beta s=0.5, \sigma x=20\%, \rho s=0, \rho fs=0.5, \rho xs=0.7 \). All curves are flat.
Quanto Adjustment (cont’d)

- Assumptions on spread distribution are important

![Graph showing the difference of Normal and Log-Normal Spread Adjustments](image)

- Difference between normal and log-normal adjustment decreases as mean reversion is increased for constant spot volatility
Conclusions

• For single-name instruments pricing is well understood

• Recovery value definition can have a significant effect on pricing and hedging

• Hedging CDS with bonds: basis risk cannot be ignored

• The distinction between EM and FI credit derivatives gradually disappears

• Consistency of pricing and hedging methods becomes more and more important