Surprising results on task assignment for high-variability workloads

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Q: What is a good Assignment Policy?

(high-variability jobs)

incoming jobs 

Poisson(λ)

Assignment Policy

FIFO

FIFO

FIFO

Goal: Minimize mean response time: \( E[T] \)

\( n \) servers

general i.i.d.

job sizes \( \sim X \)

\[ C^2 = \frac{\text{var}(X)}{E[X]^2} \]

\[ \rho = \lambda E[X] \leq n \]
Good Answers

LWL (Least Work Left)
Send job to host with least remaining work.

SITA (Size Interval)
Split jobs based on size

M/G/2

+High throughput
+Protects against high variability
+Isolation for smalls
Prior Work on SITA

**SITA in Practice**

- **Supercomputing Centers**
  - [Hotovy, Schneider, O'Donnell 96]
  - [Schroeder, Harchol-Balter 00]
- **Manufacturing Centers**
  - [Buzacott, Shanthikumar 93]
- **File Server Farms**
  - [Cardellini, Colajanni, Yu 01]
- **Supermarkets**

**SITA variants**

- [Harchol-Balter 00]
- [Harchol-Balter 02]
- [Thomas 08]
- [Tari, Broberg, Zomaya, Baldoni 05]
- [Fu, Broberg, Tari 03]

**Optimizing SITA cutoffs**

- [Harchol-Balter, Crovella, Murta 98]
- [Bachmat, Sarfati 08]
- [Sarfati 08]
- [Harchol-Balter, Vesilo 08]

**SITA vs. LWL**

- [Broberg, Tari, Zeephongsekul 06]
- [Harchol-Balter 02]
- [Ciardo, Riska, Smirni 01]
- [El-Taha, Maddah 06]
- [Fu, Broberg, Tari 03]
- [Harchol-Balter, Crovella, Murta 99]
- [Oida, Shinjo 99]
- [Tari, Broberg, Zomaya, Baldoni 05]
- [Thomas 08]

All conclude SITA far superior for high variability.
In search of a proof of SITA's total dominance.

OK, so not optimal, but definite win for high variability.

Should at least beat all commonly used policies when variability is high enough.

Can't prove anything because it's not true!
The TRUTH about SITA, under very high job size variability

\[ C^2 = \frac{\text{var}(X)}{E[X]^2} \rightarrow \infty \quad \text{while} \quad E[X]: \text{fixed} \]
Q: In this talk we will show ...

\[ C^2 \to \infty \]

a) SITA diverges & LWL diverges?
b) SITA converges & LWL diverges?
c) SITA diverges & LWL converges?
d) SITA converges & LWL converges?

A: All of the above
Q: In this talk we will show ... as $C^2 \to \infty$

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Looking for simple job size distributions to illustrate each.
Results (2 server system)

- Conv. SITA
- Diverg. SITA
- Conv. LWL
- Diverg. LWL

- Bimodal
  - p
  - 1-p

- \( \text{Exp}(\mu_a) \)
- \( \text{Exp}(\mu_b) \)

depends on \( p_a \) \& \( (1-p)b \)
Results (2 server system)

Trimodal
\[ \rho < 1 \]
\[ c = b^m > 1 \]

or

\[ H_3 \]
\[ \rho < 1 \]
\[ \text{Exp}(\mu_a) \]
\[ \text{Exp}(\mu_b) \]
\[ \text{Exp}(\mu_c) \]

Conv. SITA

Diverg. SITA

Conv. LWL

Diverg. LWL

depends on m
Results (2 server system)

- Conv. SITA
- Diverg. SITA
- Conv. LWL
- Diverg. LWL

Bounded Pareto(\(\alpha\))

\[1 < \alpha < 2\]
**Bimodal Results**

\[ X \sim p \quad \text{a} \quad (1-p) \quad \text{b} \]

\[ pa = QE[X] \]

\[ (1-p)b = (1-Q)E[X] \]

**THM**: If \( \rho_a < 1 \) & \( \rho_b < 1 \) \( \Rightarrow \) Convergent SITA

**Lemma**: As \( C^2 \to \infty \), but \( E[X], Q: \text{const} \),
- a's get little smaller \( \to QE[X] \)
- b's get much bigger \( \to \infty \)
- \( p \to 1 \)

**THM**: LWL always diverges.

**Conv. LWL**

**Conv. SITA**

**Diverg LWL**

**Diverg SITA**

depends \( \rho_a \) & \( \rho_b \)
Isn’t LWL always bad for high $C^2$?

But shorts stuck behind longs, so $E[T] \to \infty$

Need 2 longs for this to be a problem!

So we need: $\Pr\{2 \text{ longs} \} \times E[T|2 \text{ longs}]$?

Suffices to just look at $E[X^{3/2}]$. 
**Understanding LWL**

**Thm:** [Scheller-Wolf, Sigman 97], [Scheller-Wolf, Vesilo 06] (2 SERVERS)

If $E[X^{3/2}] < \infty$ \& $\rho < 1 \Rightarrow E[T]^{LWL} < \infty$

(⇐ usually)

1 spare server

$LWL \Rightarrow C^2 \rightarrow \infty$

**Thm:**

If \[
\begin{cases}
E[X^{3/2}] : \text{bounded} \\
\text{while } C^2 \rightarrow \infty
\end{cases}
\]

\& $\rho < 1 \Rightarrow \text{LWL converges}$

(⇐ usually)

I can make both happen!
**Bimodal Results**

**THM:** If $\rho_a < 1$ & $\rho_b < 1$ → Convergent SITA

- $p_a = QE[X]$
- $X \sim p \rightarrow a$
- $1-p \rightarrow b$
- $(1-p)b = (1-Q)E[X]$

**Lemma:** As $C^2 \rightarrow \infty$, but $E[X], Q$: const,
- $a \rightarrow QE[X], b \rightarrow \infty, p \rightarrow 1$

**THM:** LWL always diverges.

$$E[X^{3/2}] = pa^{3/2} + (1-p)b^{3/2} = QE[X]\sqrt{a} + (1-Q)E[X]\sqrt{b} \rightarrow \infty \ (as \ C^2 \rightarrow \infty)$$
Trimodal Results

**THM:** If $m \leq 3$, SITA converges
If $m > 3$, SITA diverges

**Lemma:** As $C^2 \to \infty$, but $E[X]$: const,
$a \to E[X]$
$b \to \infty$, $c \to \infty$
$p_a \to 1$

**THM:** LWL always converges for $\rho < 1$

$$E[X^{\frac{3}{2}}] = p_a a^{\frac{3}{2}} + p_b b^{\frac{3}{2}} + p_c c^{\frac{3}{2}}$$
$$\to E[X]^{\frac{3}{2}} + 1 + 1$$
Results
(2 server system)

Trimodal
\( \rho < 1 \)
\( c = b^m > 1 \)

Way more complex, because job types overlap!

“Separation in the limit”
Bounded Pareto
(2 server system)

\( X \sim \text{Bounded Pareto} \ (k, p, \alpha) \)

1 < \alpha < 2

Lemma: As \( C^2 \to \infty \), but \( E[X] \), \( \alpha \): const,
\[ k \to (\alpha -1)/\alpha \cdot E[X] \]
\[ p \to \infty \]

THM: SITA always diverges.

THM: If \( \alpha > 3/2 \) and \( \rho < 1 \), then LWL converges.
Else LWL diverges.

Extends to \( n > 2 \) servers when \( \rho < n - 1 \)
Bounded Pareto Results

Why was this not noticed?

$\alpha = 1.6$

$\alpha = 1.4$
Summary

Trimodal
\[ \rho < 1 \]
\[ c = b^n > 1 \]

Bimodal
\[ p \]
\[ 1 - p \]

Conv. LWL

Diverg. LWL

Conv. SITA

Diverg. SITA

or

Exp(\(\mu_a\))

H_3
\[ \rho < 1 \]
Exp(\(\mu_b\))
Exp(\(\mu_c\))

Bounded Pareto(\(\alpha\))
\[ 1 < \alpha < 2 \]
**Old Nursery Rhyme**

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When SITA is good, it is very, very good.

But when it is bad, it is horrid.